

2-3 Rules for Addition

Objective: To add real numbers using rules for addition.

Vocabulary

Opposite signs A positive and a negative number are said to have opposite signs.

Rules for Addition	Examples
If two numbers have the <i>same sign</i> , add their absolute values and put their common sign before the result.	$2 + 5 = 7$ $-2 + (-5) = -7$
If two numbers have <i>opposite signs</i> , subtract the lesser absolute value from the greater and put the sign of the number having the greater absolute value before the result.	$6 + (-4) = 6 - 4 = 2$ $(-6) + 4 = -(6 - 4) = -2$
If two numbers are <i>opposites</i> , then their sum is zero.	$3 + (-3) = 0$

Example 1 Add $6 + (-8) + 13 + (-9)$.

Solution 1 Add the numbers in order from left to right.

$$\begin{aligned} & \underline{6 + (-8)} + 13 + (-9) \\ & \underline{-2} + 13 + (-9) \\ & \underline{11} + (-9) \\ & \underline{2} \end{aligned}$$

Solution 2 1. Add positive numbers. 2. Add negative numbers. 3. Add the results.

$\begin{array}{r} 6 \\ 13 \\ \hline 19 \end{array}$	$\begin{array}{r} -8 \\ -9 \\ \hline -17 \end{array}$	$\begin{array}{r} 19 \\ -17 \\ \hline 2 \end{array}$
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Add.

1. $\begin{array}{r} 6 \\ 2 \\ \hline \end{array}$	2. $\begin{array}{r} -4 \\ -7 \\ \hline \end{array}$	3. $\begin{array}{r} -7 \\ 6 \\ \hline \end{array}$	4. $\begin{array}{r} -3 \\ 8 \\ \hline \end{array}$	5. $\begin{array}{r} 23 \\ 64 \\ \hline \end{array}$	6. $\begin{array}{r} -56 \\ 31 \\ \hline \end{array}$
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7. $\begin{array}{r} -37 \\ -56 \\ \hline \end{array}$	8. $\begin{array}{r} -35 \\ 120 \\ \hline \end{array}$	9. $\begin{array}{r} 126 \\ -35 \\ -37 \\ \hline -17 \end{array}$	10. $\begin{array}{r} -145 \\ 309 \\ -47 \\ \hline -82 \end{array}$	11. $\begin{array}{r} 136 \\ -58 \\ -47 \\ \hline -23 \end{array}$	12. $\begin{array}{r} -162 \\ 323 \\ -47 \\ \hline -82 \end{array}$
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Add.

13. $(-8 + 5) + 2$	14. $(-12 + 15) + 6$	15. $(-4 + 8) + (-3)$
16. $(-2 + 6) + (-4)$	17. $-5 + (-3) + 5$	18. $-4 + (-14) + 4$

2-3 Rules for Addition (continued)

Add.

19. $16 + 5 + (-8)$

20. $-6 + (-24) + 6$

21. $(-3 + 3) + 7 + (-11)$

22. $(-3 + 3) + 17 + (-7)$

23. $-2 + (-4) + (-8)$

24. $-7 + (-5) + (-6)$

25. $-3 + (-9) + 7 + (-5)$

26. $-15 + 10 + (-3) + (-2)$

Example 2 Simplify $3 + (-5) + (-x) + 7$.

Solution $3 + (-5) + (-x) + 7 = -x + \underbrace{3 + 7}_{10} + (-5)$ Regroup the terms.
 $= -x + \underbrace{10 + (-5)}_5$ Simplify.
 $= -x + 5$

Simplify.

evaluate $x = 2, a = 3, b = 4, y = 0, c = 2$

27. $-2 + x + (-6) + 3$

28. $3 + (-8) + (-y) + (-11)$

29. $-5 + 2a + 3 + (-3)$

30. $-5 + 2a + 8 + 7$

31. $17 + 8b + (-15) + (-10)$

32. $-[6 + (-1)] + (-c) + 2$

33. $-(-7) + 3y + (-6) + 4$

34. $3x + [7 + (-2) + (-3)]$

Example 3 Evaluate $x + y + (-2)$ if $x = -2$, and $y = 5$.

Solution $x + y + (-2) = \underbrace{(-2) + 5}_3 + (-2)$ Substitute -2 for x and 5 for y .
 $= \underbrace{3 + (-2)}_1$ Add from left to right.
 $= 1$ Simplify.

Evaluate each expression if $x = -2, y = 5$, and $z = -3$.

35. $y + z + (-2)$

36. $-18 + x + y$

37. $-11 + (-x) + (-y)$

38. $-z + (-7) + y$

39. $1 + (-y) + x$

40. $-x + (-y) + (-15)$

Mixed Review Exercises

Simplify.

1. $3 + 8 \div 2$

2. $7 \cdot 5 \cdot 3 \cdot 2$

3. $(9 - 6 \div 3) \cdot 2$

4. $|-9| - 7$

5. $|-1.6| + 1.6$

6. $|-11| - |-5|$

7. $\frac{9 \cdot 6 + 9 \cdot 4}{6 + 3}$

8. $3\frac{1}{5} + 7\frac{1}{2} + 8\frac{4}{5}$

9. $2.7 + 1.0 + 3.3$

10. $[12 + (-2)] + 5$

11. $(-7 + 2) + (-3)$

12. $-2 + (-8) + 7 + (-1)$

2-9 Dividing Real Numbers

Objective: To divide real numbers and to simplify expressions involving quotients.

Definition of Division

To divide by a nonzero real number b , multiply by the reciprocal of b .

$$a \div b \text{ or } \frac{a}{b} = a \cdot \frac{1}{b}. \quad \text{For example, } 24 \div 3 = 24 \cdot \frac{1}{3}.$$

Rules for Division

If two numbers have the same sign, their quotient is positive.

If two numbers have opposite signs, their quotient is negative.

CAUTION 1 You can't divide by zero since zero has no reciprocal.

CAUTION 2 Division is not commutative. For example, $4 \div 2 = 2$, but $2 \div 4 = \frac{1}{2}$.

CAUTION 3 Division is not associative. For example, $(12 \div 6) \div 2 = 2 \div 2 = 1$, but $12 \div (6 \div 2) = 12 \div 3 = 4$.

Example 1 Simplify: a. $\frac{30}{6}$ b. $\frac{30}{-6}$ c. $\frac{-30}{6}$ d. $\frac{-30}{-6}$

Solution a. $\frac{30}{6} = 30 \div 6 = 30 \cdot \frac{1}{6} = 5$ b. $\frac{30}{-6} = 30 \div (-6) = 30\left(-\frac{1}{6}\right) = -5$

c. $\frac{-30}{6} = -30 \div 6 = -30 \cdot \frac{1}{6} = -5$ d. $\frac{-30}{-6} = -30 \div (-6) = -30\left(-\frac{1}{6}\right) = 5$

Simplify.

1. $42 \div 14$

2. $-56 \div 7$

3. $-24 \div (-6)$

4. $-32 \div (-8)$

5. $\frac{-144}{12}$

6. $\frac{96}{-16}$

7. $\frac{-100}{-5}$

8. $\frac{-75}{-3}$

Example 2 Simplify: a. $8 \div \left(-\frac{4}{5}\right)$ b. $\frac{-4}{-\frac{1}{2}}$

Solution a. $8 \div \left(-\frac{4}{5}\right) = 8\left(-\frac{5}{4}\right) = -10$ b. $\frac{-4}{-\frac{1}{2}} = (-4) \div \left(-\frac{1}{2}\right) = (-4)(-2) = 8$

Simplify.

9. $6 \div \left(-\frac{1}{3}\right)$

10. $12 \div \left(-\frac{1}{4}\right)$

11. $0 \div \frac{5}{6}$

12. $-8 \div \left(-\frac{1}{2}\right)$

13. $0 \div \left(-\frac{2}{7}\right)$

14. $\frac{-12}{-\frac{1}{4}}$

15. $\frac{8}{-\frac{1}{2}}$

16. $\frac{-20}{\frac{1}{5}}$

17. $\frac{0}{\frac{1}{9}}$

18. $\frac{-8}{-\frac{1}{8}}$

2-9 Dividing Real Numbers (continued)**Example 3** Simplify: a. $\frac{32x}{-8}$ b. $\frac{w}{12} \cdot 12$ **Solution** a. $\frac{32x}{-8} = 32x\left(-\frac{1}{8}\right)$ Multiply by the reciprocal of -8 .

$$= 32\left(-\frac{1}{8}\right)x$$
 Regroup the factors.

$$= -4x$$
 Simplify.

b. $\frac{w}{12} \cdot 12 = w \cdot \frac{1}{12} \cdot 12$

$$= w \cdot 1$$

$$= w$$

Simplify.

19. $\frac{-18x}{3}$

20. $\frac{-42x}{6}$

21. $\frac{50x}{-10}$

22. $\frac{-36x}{-6}$

23. $5 \cdot \frac{x}{5}$

24. $\frac{-w}{8} \cdot 8$

25. $(-6)\left(\frac{-y}{2}\right)$

26. $(-10)\left(\frac{x}{-2}\right)$

27. $\frac{144b}{12}$

28. $\frac{121b}{-11}$

29. $\frac{-48x}{6}$

30. $\frac{-108x}{-36}$

Example 4 Find the average of 14, -2 , -8 , -12 .**Solution** Find the sum of the numbers and divide by the number of numbers.

$$\frac{14 + (-2) + (-8) + (-12)}{4} = \frac{-8}{4} = -2$$

Find the average of the given numbers.

31. $-12, 5, -10, -7$

32. $15, -21, -8, 6$

33. $13, -5, -16, -4$

34. $23, -13, -18, 20$

35. $11, -15, -22, 16, 0$

36. $23, -12, -17, 21, 5$

Mixed Review Exercises**Solve if $x \in \{0, 1, 2, 3, 4, 5, 6\}$.**

1. $x + 5 = 7$

2. $\frac{1}{2}x = 3$

3. $x - 1 = 4$

4. $3x = 9$

5. $3x + 1 = 7$

6. $x \div 3 = 1$

Solve over the domain $\{0, 1, 2, 3, 4, 5\}$.

7. $\frac{1}{3}n = 1$

8. $3y - 1 = 14$

9. $x + 2 = 6$

10. $2x = 2$

11. $x \cdot x = 4$

12. $3n = n \cdot 3$

3 Solving Equations and Problems

3-1 Transforming Equations: Addition and Subtraction

Objective: To solve equations using addition and subtraction.

Properties

Addition Property of Equality If the same number is added to equal numbers, the sums are equal.

Subtraction Property of Equality If the same number is subtracted from equal numbers, the differences are equal.

Vocabulary

Equivalent equations Equations that have the same solution set over a given domain.

Transformations Operations on an equation that produce a simpler equivalent equation.

By substitution You can substitute an equivalent expression for any expression in an equation. You do this when you simplify an expression in an equation.

By addition You can add the same number to each side of an equation.

By subtraction You can subtract the same number from each side of an equation.

CAUTION

To check your work, you should check that each solution of the final equation satisfies the *original equation*.

Example 1 Solve $x - 6 = 11$.

Solution

$$x - 6 = 11$$

$$x - 6 + 6 = 11 + 6$$

$$x = 17$$

Check: $x - 6 = 11$ ← Original equation.

$$17 - 6 \stackrel{?}{=} 11$$

$$11 = 11 \checkmark$$

Substitute 17 for x .

The solution set is $\{17\}$.

Solve.

1. $a - 9 = 11$

2. $b - 5 = 13$

3. $x - 20 = -19$

4. $d - 14 = 5$

5. $x - 15 = 0$

6. $v - 27 = -54$

7. $x - 6 = 27$

8. $q - 7 = 11$

9. $q - 9 = -16$

3-1 Transforming Equations: Addition and Subtraction (continued)**Example 2** Solve $-9 = n + 11$.**Solution**

$$-9 = n + 11$$

$$-9 - 11 = n + 11 - 11$$

$$-20 = n$$

{ To get n alone on one side,
subtract 11 from each side.
Simplify.

Check: $-9 = n + 11$ ← Original equation

$$-9 \stackrel{?}{=} -20 + 11$$

$$-9 = -9 \checkmark$$

Substitute -20 for n .The solution set is $\{-20\}$.**Solve.**

10. $-6 = m + 6$

11. $21 = x + 15$

12. $-26 + m = 24$

13. $-37 + n = 63$

14. $p + 18 = -22$

15. $a + 60 = -15$

16. $14 + t = 0$

17. $29 = y - 12$

18. $35 = x + 16$

19. $-4 = u - 6$

20. $22 = y + 3$

21. $c - 8 = -10$

22. $x + 1.5 = 6.8$

23. $-1 + a = 0.5$

24. $3.9 = y - 1.4$

25. $7.5 = w - 2.5$

Example 3 Solve $-x + 5 = 4$.**Solution**

$$-x + 5 = 4$$

$$-x + 5 - 5 = 4 - 5$$

$$-x = -1$$

$$x = 1$$

{ To get x alone on one side,
subtract 5 from each side and simplify.
If the opposite of a number is -1 ,
the number must be 1.

Check: $-x + 5 = 4$ ← Original equation

$$-1 + 5 \stackrel{?}{=} 4$$

$$4 = 4 \checkmark$$

Substitute 1 for x .The solution set is $\{1\}$.**Solve.**

26. $-x + 3 = 5$

27. $-y + 7 = 17$

28. $12 - x = 18$

29. $7 - y = 11$

30. $9 = -x + 16$

31. $13 = 22 - y$

32. $-5 - y = 7$

33. $10 = -12 - e$

34. $15 = -y + 10$

Mixed Review ExercisesEvaluate if $a = 3$, $b = -6$, $c = -4$, and $d = 2$.

1. $a - |b - c|$

2. $(|c| - d) - (|b| - a)$

3. $3|c| - (-b)$

4. $\frac{a - 2b}{a + d}$

5. $\frac{3b + c - d}{ad}$

6. $\frac{2ab}{c + d}$

Simplify.

7. $(-3)(-4)(8)$

8. $(-7 \cdot 16) + (-7 \cdot 24)$

9. $252 \div (-36)$

3-2 Transforming Equations: Multiplication and Division

Objective: To solve equations using multiplication or division.

Properties

Multiplication Property of Equality If equal numbers are multiplied by the same number, the products are equal.

Division Property of Equality If equal numbers are divided by the same *nonzero* number, the quotients are equal.

Transformations

By multiplication You can multiply each side of an equation by the same *nonzero* real number.

By division You can divide each side of an equation by the same *nonzero* real number.

CAUTION 1 When you transform an equation, never multiply or divide by zero.

CAUTION 2 When you multiply or divide by a negative number, be careful with the sign of your answer.

Example 1 Solve $4x = 128$.

Solution

$$\begin{aligned}\frac{4x}{4} &= \frac{128}{4} \\ x &= 32\end{aligned}$$

$$\begin{aligned}\text{Check: } 4x &= 128 \\ 4(32) &\stackrel{?}{=} 128 \\ 128 &= 128 \checkmark\end{aligned}$$

To get x alone on one side, divide each side by 4 (or multiply by $\frac{1}{4}$, the reciprocal of 4).

The solution set is $\{32\}$.

Solve.

1. $7m = 140$

2. $12n = 240$

3. $-8x = 96$

4. $-11f = -143$

5. $-720 = -24g$

6. $330 = -15u$

7. $108 = -9x$

8. $45k = -270$

9. $26n = -520$

Example 2 Solve $12 = -\frac{3}{4}n$.

Solution

$$\begin{aligned}-\frac{4}{3}(12) &= -\frac{4}{3}\left(-\frac{3}{4}n\right) \\ -16 &= n\end{aligned}$$

$$\begin{aligned}\text{Check: } 12 &= -\frac{3}{4}n \\ 12 &\stackrel{?}{=} -\frac{3}{4}(-16) \\ 12 &= 12 \checkmark\end{aligned}$$

To get n alone on one side, multiply each side by $-\frac{4}{3}$, the reciprocal of $-\frac{3}{4}$.

The solution set is $\{-16\}$.

3-2 Transforming Equations: Multiplication and Division (continued)

Solve.

10. $\frac{2}{3}m = 6$

11. $\frac{3}{5}y = -15$

12. $-\frac{5}{8}x = 40$

13. $-\frac{4}{5}y = -20$

14. $\frac{2}{5}d = -40$

15. $\frac{3}{4}g = -24$

16. $\frac{7}{8}y = -56$

17. $-\frac{7}{10}e = 140$

18. $-\frac{2}{7}n = -28$

Example 3 Solve: a. $\frac{x}{2} = -6$

b. $\frac{1}{2}n = 3\frac{1}{2}$

Solution

$$2\left(\frac{x}{2}\right) = 2(-6)$$

$$x = -12$$

Check: $\frac{x}{2} = -6$

$$\frac{-12}{2} \stackrel{?}{=} -6$$

$$-6 = -6 \checkmark$$

The solution set is $\{-12\}$.

$$\frac{1}{2}n = \frac{7}{2}$$

$$2\left(\frac{1}{2}\right)n = 2\left(\frac{7}{2}\right)$$

$$n = 7$$

Check: $\frac{1}{2}n = 3\frac{1}{2}$

$$\frac{1}{2}(7) \stackrel{?}{=} 3\frac{1}{2}$$

$$\frac{7}{2} = 3\frac{1}{2} \checkmark$$

The solution set is $\{7\}$.

Solve.

19. $\frac{c}{6} = -24$

20. $\frac{y}{5} = -25$

21. $-\frac{u}{12} = 12$

22. $-\frac{x}{3} = 15$

23. $-28 = \frac{c}{7}$

24. $-\frac{1}{4}x = 2\frac{1}{4}$

25. $\frac{1}{5}f = 3\frac{1}{5}$

26. $\frac{1}{2}b = 2\frac{1}{2}$

27. $-\frac{1}{3}y = 3\frac{2}{3}$

Mixed Review ExercisesEvaluate if $a = -2$, $b = 3$, and $c = -6$.

1. $6b - 2a$

2. $(2b - 5c)a$

3. $|c| + |a| - b$

4. $|b| - |a + c|$

5. $\frac{-(7ab)}{c}$

6. $\frac{8 + a}{c}$

Simplify.

7. $6a + 5 + 7a$

8. $7n - 6 + 6$

9. $9p - p + 3$

10. $-3(m + 4)$

11. $(x + 5)6$

12. $2(3y - 4)$