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5–7 Factoring Pattern for $x^2 + bx + c$, c positive

Objective: To factor quadratic trinomials whose quadratic coefficient is 1 and whose constant term is positive.

Vocabulary/Patterns

Factoring patterns for $x^2 + bx + c$ when c is positive:
When b is positive: $(x + ?)(x + ?)$
When b is negative: $(x - ?)(x - ?)$

Prime polynomial A polynomial with integral coefficients whose greatest monomial factor is 1 and which can't be written as a product of polynomials of lower degree. For example, $a^2 - 10a - 14$ is prime.

Example 1 Factor $x^2 + 6x + 8$.

Solution

1. The coefficient of the linear term is positive. The pattern is $(x + ?)(x + ?)$. List the positive factors of 8.

Factors of 8		Sum of the factors
1	8	9
2	4	6 ←

2. Find the pair of factors whose sum is 6: 4 and 2.
3. Therefore $x^2 + 6x + 8 = (x + 4)(x + 2)$.

You can check the result by multiplying $(x + 4)$ and $(x + 2)$.

$(x + 4)(x + 2) = x^2 + 2x + 4x + 8 = x^2 + 6x + 8$ √

Example 2 Factor $x^2 - 8x + 15$.

Solution

1. The coefficient of the linear term is negative. The pattern is $(x - ?)(x - ?)$ List the pairs of negative factors of 15.

Factors of 15		Sum of the factors
−1	−15	−16
−3	−5	−8 ←

2. Find the pair of factors whose sum is −8: −3 and −5.
3. Therefore $x^2 - 8x + 15 = (x - 3)(x - 5)$.

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime.*

1. $x^2 + 4x + 3$
2. $x^2 + 8x + 7$
3. $c^2 - 9c + 14$
4. $y^2 - 8y + 12$
5. $r^2 - 5r + 6$
6. $p^2 - 13p + 12$
7. $q^2 + 15q + 14$
8. $n^2 + 9n + 14$
9. $a^2 - 13a + 22$
10. $s^2 - 12s + 30$
11. $x^2 + 18x + 32$
12. $x^2 - 15x + 26$

Example 3 Factor $y^2 - 10y + 16$.

Solution

1. Since -10 is negative, think of the negative factors of 16 in your head. (After a little practice you will not need to write all the factors down.)
2. Select the factors of 16 with sum -10: -2 and -8.
3. Therefore $y^2 - 10y + 16 = (y - 2)(y - 8)$.

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

13. $a^2 + 10a + 30$
14. $x^2 - 19x + 60$
15. $k^2 - 21k + 54$
16. $n^2 + 23n + 90$
17. $k^2 - 10k + 21$
18. $x^2 - 14x + 45$
19. $k^2 + 7k + 12$
20. $x^2 - 16x + 48$
21. $a^2 - 11a + 20$
22. $x^2 + 22x + 72$
23. $72 - 17z + z^2$
24. $20 - 12c + c^2$
25. $54 - 15a + a^2$
26. $63 - 16c + c^2$

Example 4 Factor $x^2 - 12xy + 32y^2$.

Solution

$x^2 - 12xy + 32y^2 = (x - ?)(x - ?)$ Write the factoring pattern.

$= (x - 4y)(x - 8y)$ Fill in the negative factors of $32y^2$.

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

27. $x^2 - 11xy + 28y^2$
28. $a^2 - 9ab + 18b^2$
29. $c^2 - 18cd + 45d^2$
30. $x^2 - 10xy + 21y^2$
31. $c^2 - 14cd + 24d^2$
32. $x^2 + 11xy + 30y^2$
33. $y^2 - 16yz + 48z^2$
34. $a^2 - 18ab + 45b^2$
35. $d^2 + 10de + 24e^2$
36. $y^2 - 27yz + 72z^2$

Mixed Review Exercises

Solve.

1. $-12 + x = -7$
2. $d + (-4) = -9$
3. $-12 + b = 13$
4. $a + 3 = |2 - 9|$
5. $17m = 68$
6. $3p + 15 = -60$
7. $-\frac{1}{3}x = 9$
8. $\frac{r}{2} - 3 = 6$
9. $-18x = 162$

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5–8 *Factoring Pattern for $x^2 + bx + c$, c negative*

Objective: To factor quadratic trinomials whose quadratic coefficient is 1 and whose constant term is negative.

Patterns

Factoring pattern for $x^2 + bx + c$ when c is negative: $(x + ?)(x - ?)$

Example 1 Factor $x^2 - x - 12$.

Solution

1. List the factors of -12 by writing them down or reviewing them mentally.
2. Find the pair of factors with sum -1: 3 and -4.
3. Therefore $x^2 - x - 12 = (x + 3)(x - 4)$.

Factors of −12		Sum of the factors
1	−12	−11
−1	12	11
2	−6	−4
−2	6	4
3	−4	−1 ←
−3	4	1

Example 2 Factor $a^2 + 12a - 45$.

Solution

1. The factoring pattern is $(a + ?)(a - ?)$.
2. Find the pair of factors of -45 whose sum is 12: 15 and -3.
3. Therefore $a^2 + 12a - 45 = (a + 15)(a - 3)$.

Factors of −45		Sum of the factors
1	−45	−44
−1	45	44
3	−15	−12
−3	15	12 ←
5	−9	−4
−5	9	4

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

1. $a^2 - 2a - 3$
2. $x^2 + x - 6$
3. $y^2 + 3y - 4$
4. $b^2 - 3b - 10$
5. $c^2 - 9c - 8$
6. $r^2 + 12r - 28$
7. $x^2 - 7x - 18$
8. $y^2 + 4y - 21$
9. $a^2 + 5a - 14$
10. $k^2 - 6k - 40$
11. $z^2 + 6z - 27$
12. $r^2 - 2r - 35$
13. $p^2 - 4p - 12$
14. $a^2 - 3a - 40$
15. $y^2 - 8y - 20$
16. $z^2 - z - 56$
17. $y^2 - 14y - 72$
18. $t^2 + 16t - 30$

5–8 Factoring Pattern for $x^2 + bx + c$, c negative *(continued)*

Example 3 Factor $x^2 - 5kx - 24k^2$.

Solution
1. The factoring pattern is $(x + ?)(x - ?)$.
2. Find the pair of factors of $-24k^2$ with a sum of $-5k$: $3k$ and $-8k$.
3. Therefore $x^2 - 5kx - 24k^2 = (x + 3k)(x - 8k)$.

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

19. $a^2 - ab - 20b^2$ **20.** $y^2 - yz - 12z^2$ **21.** $u^2 - 3uv - 18v^2$

22. $a^2 - 5ab - 24b^2$ **23.** $x^2 - 7xy - 30y^2$ **24.** $h^2 - 2hk - 24k^2$

25. $x^2 + 5xy - 50y^2$ **26.** $c^2 - 2cd - 35d^2$ **27.** $x^2 - 11xy - 42y^2$

Example 4 Factor $1 - 8x - 20x^2$.

Solution Find the pair of factors of $-20x^2$ whose sum is $-8x$: $2x$ and $-10x$.
$1 - 8x - 20x^2 = (1 + 2x)(1 - 10x)$

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

28. $1 + 2c - 24c^2$ **29.** $1 + 9c - 36c^2$ **30.** $1 + 5x - 24x^2$

31. $1 + 5x - 36x^2$ **32.** $1 - 14y - 72y^2$ **33.** $1 - 12x - 45x^2$

34. $1 - 4x - 21x^2$ **35.** $1 - 7x - 30x^2$ **36.** $1 + 7x - 44x^2$

Mixed Review Exercises

Simplify.

1. $(8x^2y)(4xy^2)(3y^2)$ **2.** $(3x - 2)(2x + 3)$ **3.** $-5x(2x^2 - x + 3)$

4. $(2x - 3)^2$ **5.** $(5x^4y^2)^3$ **6.** $4y(2y^2 + 5y + 3)$

7. $\frac{4(xy)^4}{8(xy)^2}$ **8.** $\frac{-3ab}{-18a^2b^3}$ **9.** $\frac{(-n)^4}{-n^8}$

10. $(m + 2n)^2$ **11.** $(a - 4)(3a + 2)$ **12.** $(2y + 5)^2$

Factor.

13. $10m - 14n + 2$ **14.** $81k^2 - 25$ **15.** $a^2 + 8a + 16$

16. $a^2 - 11ab + 24b^2$ **17.** $18x^2 + 12x$ **18.** $49 - n^2$

19. $u^2 - 18u + 81$ **20.** $27 + 12y + y^2$ **21.** $6a^3b^2 - 18a^2b$

22. $25w^4 - 9x^2$ **23.** $m^2 + 3m + 2$ **24.** $c^2 - 9c - 22$

5–9 Factoring Pattern for $ax^2 + bx + c$

Objective: To factor general quadratic trinomials with integral coefficients.

Patterns

Factoring pattern for $ax^2 + bx + c$: $(px + r)(qx + s)$.

Example 1 Factor $2x^2 - 3x - 9$.

Solution

Clue 1 Because the trinomial has a negative constant term, one of r and s will be negative and the other will be positive.

Clue 2 You can list the possible factors of the quadratic term, $2x^2$, and the possible factors of the constant term, -9.

Factors of $2x^2$	Factors of -9	
$2x, x$	$1, -9$	$-1, 9$
	$3, -3$	$-3, 3$
	$9, -1$	$-9, 1$

Make a chart to test the possibilities to see which produces the correct linear term, $-3x$.

Since $(2x + 3)(x - 3)$ gives the correct linear term, $2x^2 - 3x - 9 = (2x + 3)(x - 3)$.

Possible factors	Linear Term
$(2x + 1)(x - 9)$	$(-18 + 1)x = -17x$
$(2x + 3)(x - 3)$	$(-6 + 3)x = -3x$ ←
$(2x + 9)(x - 1)$	$(-2 + 9)x = 7x$
$(2x - 1)(x + 9)$	$(18 - 1)x = 17x$
$(2x - 3)(x + 3)$	$(6 - 3)x = 3x$
$(2x - 9)(x + 1)$	$(2 - 9)x = -7x$

Example 2 Factor $10x^2 - 11x + 3$.

Solution

Clue 1 Because the trinomial has a positive constant term and a negative linear term, both r and s will be negative.

Clue 2 List the factors of the quadratic term, $10x^2$, and the negative factors of the constant term, 3.

Factors of $10x^2$	Factors of 3
$x, 10x$	$-3, -1$
$2x, 5x$	$-1, -3$

Test the possibilities to see which produces $-11x$. Since $(2x - 1)(5x - 3)$ gives the correct linear term, $10x^2 - 11x + 3 = (2x - 1)(5x - 3)$.

Possible factors	Linear term
$(x - 3)(10x - 1)$	$(-1 - 30)x = -31x$
$(x - 1)(10x - 3)$	$(-3 - 10)x = -13x$
$(2x - 3)(5x - 1)$	$(-2 - 15)x = -17x$
$(2x - 1)(5x - 3)$	$(-6 - 5)x = -11x$ ←

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

1. $2x^2 + 5x + 2$ **2.** $2n^2 - 7n + 3$ **3.** $5y^2 - 9y - 2$ **4.** $3a^2 + 7a + 2$

5. $4y^2 - 5y + 1$ **6.** $2a^2 + 11a + 5$ **7.** $5a^2 - 11a + 2$ **8.** $7y^2 - 9y + 2$

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Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

9. $2k^2 - 5k - 1$ **10.** $12k^2 - 8k + 1$ **11.** $4x^2 + 17x - 15$ **12.** $2a^2 + 7a + 5$

13. $8y^2 + 6y - 9$ **14.** $9x^2 + 3x - 2$ **15.** $7k^2 - 11k - 6$ **16.** $4u^2 - 8u - 5$

Example 3 Factor $5 - 7x - 6x^2$.

Solution $5 - 7x - 6x^2 = -6x^2 - 7x + 5$ Arrange the terms by decreasing degree.
$= (-1)(6x^2 + 7x - 5)$ Factor -1 from each term.
$= (-1)(2x - 1)(3x + 5)$ Factor the resulting trinomial.
$= -(2x - 1)(3x + 5)$

Note: If you factor $5 - 7x - 6x^2$ directly, you will get $(5 + 3x)(1 - 2x)$. Since $(1 - 2x) = -(2x - 1)$, the two answers are equivalent.

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

17. $10 - 9y - 2y^2$ **18.** $10 - x - 3x^2$ **19.** $3 - x - 10x^2$

20. $3 - 7x - 6x^2$ **21.** $10 - u - 2u^2$ **22.** $5 + 8x - 4x^2$

Example 4 Factor $5a^2 + 2ab - 7b^2$.

Solution $5a^2 + 2ab - 7b^2 = (a \quad)(5a \quad)$ Write the factors of $5a^2$.
$= (a - ?)(5a + ?)$ Test possibilities.
$= (a - b)(5a + 7b)$

Note: If you write $(a + ?)(5a - ?)$ as the second step, you will not find a combination of factors that produces the desired linear term.

Factor. Check by multiplying the factors.

23. $x^2 - xy - 20y^2$ **24.** $4a^2 - 4ab - 3b^2$ **25.** $3a^2 - 5ab - 12b^2$

26. $5a^2 + 2ab - 7b^2$ **27.** $2x^2 - xy - 3y^2$ **28.** $8y^2 - 6yz - 9z^2$

Mixed Review Exercises

Factor.

1. $x^2 - 196$ **2.** $x^2 - 7x + 12$ **3.** $r^2 - 5r - 36$

4. $c^2 - 10c + 25$ **5.** $9y^2 - 121x^2$ **6.** $4a^2 - 25$

7. $y^2 + 13y + 36$ **8.** $p^2 + 14p + 49$ **9.** $9y^2 + 12y + 4$

10. $m^2 - m - 56$ **11.** $n^2 + 13n + 36$ **12.** $b^2 - 3b - 54$